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Skin Depth of Electromagnetic Waves in Plasma with Magnetic Field and Collisions

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Skin depth δ and wave number of various electromagnetic waves in plasma in which collisions occur have been analyzed without and with the magnetic field: right (R)- and left (L)- hand circularly polarized, ordinary (O) and extraordinary (X) waves. For most evanescent waves, δ is expressed as $fc/\omega_{\rm pe}$ for $\omega_{\rm pe} \gg \omega$, ν (f: collision term, ν : collision frequency, c : velocity of light, $\omega_{\rm pe}$: electron plasma angular frequency). For L and X waves under limited conditions, δ is $c/\omega_{\rm pe}$ multiplied by $(\omega_{\rm ce}/\omega)^{0.5}$ and $\omega_{\rm ce}/\omega_{\rm pe}$, respectively ($\omega_{\rm ce}$: electron cyclotron angular frequency). In the low ν range with $\omega \approx \omega_{\rm pe}$, δ becomes large for light, O, R and X waves ($\omega \gg \omega_{\rm ce}$).

KEYWORDS: plasma, skin depth, collision, R wave, L wave, ordinary wave, extraordinary wave, magnetic field, electric field

The understanding of the penetration length, i.e. a skin depth, of the electromagnetic field in a plasma is very important, since this length is related to the heating layer and heating mechanism. The collisionless (classical) skin depth is known as $c/\omega_{\rm pe}$ without static magnetic field for the case of $\omega \ll \omega_{\rm pe}$ (c: velocity of light, $\omega_{\rm pe}$: electron plasma angular frequency, ω: excited angular frequency). This length is also related to interesting phenomena such as the anomalous skin effect, which is considered to arise due to thermal electron motion[1)] and some instabilities, *e.g.*, ion acoustic instability[2)], and the nonlinear skin effect[3)] due to a large-amplitude electromagnetic wave. In addition, effective RF heating in capacitively coupled plasma (CCP), which is related to a heating phenomenon in the RF sheath region[4)], and in inductively coupled plasma (ICP)[5–7)] even under low collisionality, and turbulent heating in tokamaks[8)] as well as heating by magnetic surface waves[9)] require clarification of the heating mechanism associated with the skin layer. However, basic studies, as well as experiments, on the evaluation of this skin layer in the presence of a static magnetic field with a collision effect in the linear wave regime are incomplete and have rarely been reported[10)] in spite of the importance of a comparative study between experiment and theory.

Here, we derive analytic formulae of the skin depth δ and real wave number $k_{\rm r}$ of the electromagnetic evanescent waves in the plasma from a cold dispersion relation, taking collision into account. A collisional damping length $L_{\rm d}$ and real wave number $k_{\rm r}$ are also presented for propagating waves. These formulae have been obtained for cases without (light wave) and with the static straight magnetic field: parallel (right-hand (R) and left-hand (L) circularly polarized waves) and perpendicular (ordinary (O) and extraordinary (X) waves) wave propagation. In addition, numerical results of skin depth for those types of evanescent waves have been shown. Note that the expressions derived hold as long as the electron thermal motion is small ($c/\omega_{\rm pe} \geq v_{\rm the}/\omega$ ($v_{\rm the}$: electron thermal velocity)) and / or collision frequency ν is high ($\nu \gg \omega$), which is satisfied under most CCP and ICP experimental conditions, for the case of the linear wave regime without instabilities.

Hereafter, we use the cold plasma dispersion relation[11,12)] assuming $m_{\rm i}$ (mass of ion) $\gg$ $m_{\rm e}$ (mass of electron) and a two-component plasma of electrons and ions (single species). In the absence of magnetic field, the dispersion relation of an electromagnetic wave (light wave) with a collision is written as,

$$\frac{k_{\parallel}^2 c^2}{\omega^2} = 1 - \frac{\omega_{\rm pe}^2}{\omega^2(1+{\rm i}\nu^*)}, \tag{1}$$

where $k_{\parallel}$ is a parallel wave number. In general, $m_{\rm e}$ in $\omega_{\rm pe}$ and $\omega_{\rm ce}$ (electron cyclotron angular frequency) may be replaced by $(1+{\rm i}\nu/\omega)m_{\rm e}$ in the presence of an electron collision[11)]. Exact solutions of $k_{\rm r}$ (upper sigh) and $k_{\rm i}$ (lower sign), which are real and imaginary parts of $k_{\parallel}$, respectively, from eq. (1) are[13)]

$$k_{\rm r,i}^2 = \frac{\omega}{2c^2(\omega^2+\nu^2)}\{\pm\omega(\omega^2+\nu^2-\omega_{\rm pe}^2) + [(\omega^2+\nu^2-\omega_{\rm pe}^2)^2\omega^2+\omega_{\rm pe}^4\nu^2]^{0.5}\}. \tag{2}$$

When $\omega_{\rm pe} \gg \omega$ (nonpropagating region) and $\omega_{\rm pe} \gg \nu$ are satisfied, the approximate expressions of skin depth $\delta = 1/k_{\rm i}$ and real parallel wave number $k_{\rm r}$ from eq. (2), using a normalized collision frequency $\nu^* = \nu/\omega$, become

$$\delta = (c/\omega_{\rm pe})f(\nu^*),\ k_{\rm r} = (\omega_{\rm pe}/c)g(\nu^*),$$
$$f(\nu^*) = \left[\frac{2(1+\nu^{*2})}{1+(1+\nu^{*2})^{0.5}}\right]^{0.5},$$
$$g(\nu^*) = \frac{\nu^*}{\{2(1+\nu^{*2})[1+(1+\nu^{*2})^{0.5}]\}^{0.5}}. \tag{3}$$

From these formulae, skin depth δ is different from collisionless skin depth $\delta_{\rm cl} = c/\omega_{\rm pe}$ (for $\omega_{\rm pe} \gg \omega$) by a factor of $f(\nu^*)$, which is nearly unity at small ν^* and becomes large at large ν^* (approximate form of $f(\nu^*)$ is $(2\nu^*)^{0.5}$). The real wave number $k_{\rm r}$ is $1/\delta_{\rm cl}$ multiplied by $g(\nu^*)$, which increases with ν^*. The ratio of $1/k_{\rm r}$ to δ, i.e., $k_{\rm i}/k_{\rm r}$, is $1/f(\nu^*)g(\nu^*) = [1+(1+\nu^{*2})^{0.5}]/\nu^*$. These dependences of $f(\nu^*)$, $g(\nu^*)$ and $1/f(\nu^*)g(\nu^*)$ on ν^* are shown in Fig. 1. Note that ω approaches $\omega_{\rm pe}$ ($\Delta\omega = \omega_{\rm pe} - \omega > 0$), δ increases and δ and $k_{\rm r}$ are scaled as

$$\delta = (c/\omega_{\rm pe})[2(1+\nu^{*2})/h]^{0.5},$$
$$k_r = (\omega_{\rm pe}/c)\{\nu^*/[2(1+\nu^{*2})]^{0.5}\}/h^{0.5},$$

respectively, where

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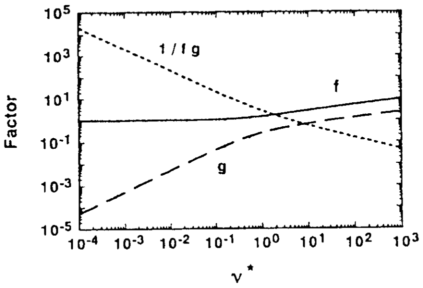

Fig. 1. Dependences of $f(\nu^*)$, $g(\nu^*)$ and $1/f(\nu^*)g(\nu^*)$ on the normalized collision frequency $\nu^* = \nu/\omega$ for the light and ordinary (O) waves with $\omega_{\rm pe} \gg \omega, \nu$. Here, skin depth and real wave number can be written as $\delta = \delta_{cl} f(\nu^*)$ and $k_{\rm r} = (1/\delta_{cl})g(\nu^*)$, respectively, and the ratio of $1/k_{\rm r}$ to δ is $1/f(\nu^*)g(\nu^*)$ (δ_{cl}: classical collisionless skin depth of $c/\omega_{\rm pe}$).

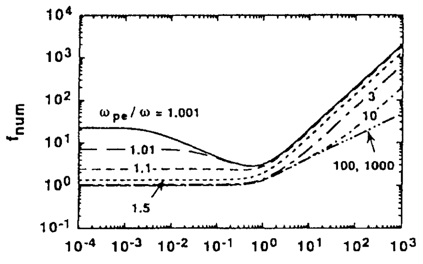

Fig. 2. Relationship between enhancement factor $f_{\rm num}$ of the skin depth and normalized collision frequency ν^* for various values of $\omega_{\rm pe}/\omega$ for light and ordinary (O) waves.

$$h = [2(1+\nu^{*2})\Delta\omega/\omega_{\rm pe} - \nu^{*2}] + \{[2(1+\nu^{*2})\Delta\omega/\omega_{\rm pe} - \nu^{*2}]^2 + \nu^{*2}\}^{0.5}. \quad (4)$$

Further approximate forms of δ and $k_{\rm r}$ are $\delta = (c/\omega_{\rm pe})(\omega_{\rm pe}/2\Delta\omega)^{0.5}$ and $k_{\rm r} = 0$ for $\nu^* \approx 0$, and $\delta = (c/\omega_{\rm pe})\{2(1+\nu^{*2})/\nu^*[-\nu^* + (1+\nu^{*2})^{0.5}]\}^{0.5}$ and $k_{\rm r} = (\omega_{\rm pe}/c)\{\nu^*/2(1+\nu^{*2})[-\nu^* + (1+\nu^{*2})^{0.5}]^{0.5}\}^{0.5}$ for $\Delta\omega/\omega_{\rm pe} \approx 0$. For $\Delta\omega/\omega_{\rm pe} \ll \nu^{*2} \ll 1$, δ and $k_{\rm r}$ can be simplified as $\delta = (c/\omega_{\rm pe})(2/\nu^*)^{0.5}$ and $k_{\rm r} = (\omega_{\rm pe}/c)(2\nu^*)^{0.5}$. When a wave propagates with $\omega \gg \omega_{\rm pe}$, wave number $k_{\rm r}$ becomes ω/c and there is a long length of a collisional damping: $L_{\rm d} = 1/k_{\rm i} = (2c\omega/\omega_{\rm pe}^2)[(1+\nu^{*2})/\nu^*]$.

Figure 2 shows the numerical result using eq. (2) for the relation between enhancement factor $f_{\rm num}$, namely, $\delta = (c/\omega_{\rm pe})f_{\rm num}$, and normalized collision frequency ν^* for various values of $\omega_{\rm pe}/\omega$. As will be mentioned later, this result for this light wave can also be applied to the ordinary (O) wave. From this figure, it is clear that $f_{\rm num}$ becomes large when ω is close to $\omega_{\rm pe}$ from the lower frequency side with a small ν^* value which has been analytically derived as $(2/\nu^*)^{0.5}$ and exhibits a propagating wave ($\omega_{\rm pe}/\omega < 1$) character. For high $\omega_{\rm pe}/\omega$ with a moderate ν^* value, $f_{\rm num}$ becomes nearly the same as $f(\nu^*)$ in Fig. 1.

Next, wave propagation along the static uniform magnetic field is considered. The dispersion relations of the R (upper sign) and L (lower sign) waves without a collision are

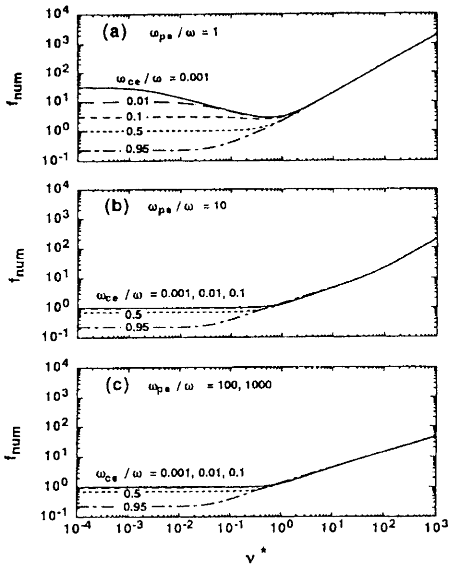

Fig. 3. Relationship between enhancement factor $f_{\rm num}$ of the skin depth and normalized collision frequency ν^* for various values of $\omega_{\rm pe}/\omega$ and $\omega_{\rm ce}/\omega$ for right-hand circularly polarized (R) wave.

$$\frac{k_\parallel^2 c^2}{\omega^2} = \frac{(\omega \mp \omega_R)(\omega \pm \omega_L)}{(\omega \pm \omega_{\rm ci})(\omega \mp \omega_{\rm ce})}, \quad (5)$$

respectively, where $\omega_{\rm ci}$ is an ion cyclotron angular frequency, and $\omega_{\rm R}$ (upper sign) and $\omega_{\rm L}$ (lower sign) are written as

$$\omega_{\rm R,L} = \pm\frac{\omega_{\rm ce}}{2} + \left[\left(\frac{\omega_{\rm ce}}{2}\right)^2 + \omega_{\rm pe}^2 + \omega_{\rm ce}\omega_{\rm ci}\right]^{0.5}, \quad (6)$$

respectively. When $\omega_{\rm pe} \gg \omega_{\rm ce}$ is satisfied, ω_R and ω_L are close to $\omega_{\rm pe}$. From eq. (5), using $\omega \gg \omega_{\rm ci}$, the dispersion relations of the electron cyclotron wave (R wave, upper sign) and L wave (lower sign) are modified as

$$\frac{k_\parallel^2 c^2}{\omega^2} = 1 - \frac{\omega_{\rm pe}^2}{\omega(\omega \mp \omega_{\rm ce})}, \quad (7)$$

respectively.

If the condition of $\omega_{\rm R} \gg \omega \gg \omega_{\rm ce}$ is added and the collision effect is included for the R wave, $k_\parallel$ derived from eq. (5) becomes approximately the same as eq. (1), which makes $k_{\rm i}$, δ and $k_{\rm r}$ the same as in eqs. (2) and (3). Note that the nonpropagating ω region ($\omega_{\rm R} > \omega > \omega_{\rm ce}$) is narrow for the case of $\omega_{\rm ce} \gg \omega_{\rm pe}$. When ω is close to $\omega_{\rm ce}$ (i.e. electron cyclotron resonance condition) from the higher frequency side ($\Delta\omega = \omega - \omega_{\rm ce} > 0$), δ becomes small of $(c/\omega_{\rm pe})(\Delta\omega/\omega_{\rm ce})^{0.5}$ for $\nu^* \approx 0$.

Figure 3 shows the numerical result for the dependence of enhancement factor $f_{\rm num}$ on the normalized collision frequency ν^* in the evanescent region of the R wave, obtained using eq. (7). For low $\omega_{\rm ce}/\omega$ (< 0.5) with $\omega_{\rm pe}/\omega = 10$ and 100, $f_{\rm num}$ (≈ 1) is nearly independent of ν^* in the low ν^* region, and $f_{\rm num}$ becomes larger for lower $\omega_{\rm pe}/\omega$ ($= 1$). In the high ν^* region, for a given $\omega_{\rm pe}/\omega$ (regardless of $\omega_{\rm ce}/\omega$), $f_{\rm num}$ is nearly the same as $f(\nu^*)$, which is shown in Fig. 2. For the case that $\omega_{\rm ce}/\omega$ is close to 1 ($\omega > \omega_{\rm ce}$), where the wave is evanescent but near an electron cyclotron resonance condition, $f_{\rm num}$ is less than unity ($(\Delta\omega/\omega_{\rm ce})^{0.5}$ has been analytically derived).

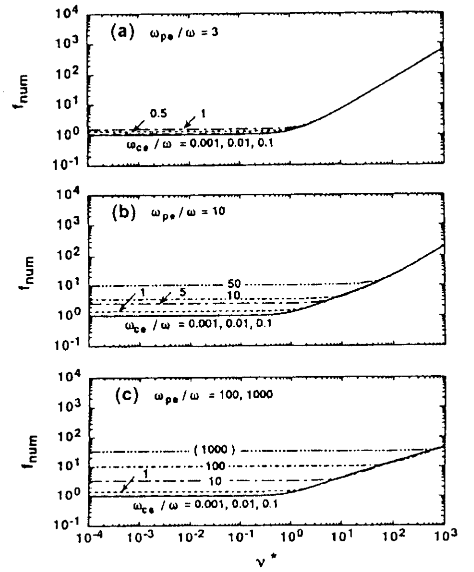

Fig. 4. Relationship between enhancement factor $f_{\rm num}$ of the skin depth and normalized collision frequency ν^* for various values of $\omega_{\rm pe}/\omega$ and $\omega_{\rm ce}/\omega$ for left-hand circularly polarized (L) wave.

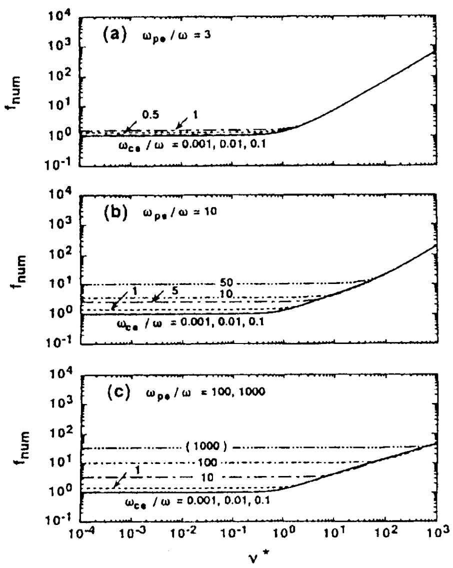

Fig. 5. Relationship between enhancement factor $f_{\rm num}$ of the skin depth and normalized collision frequency ν^* for various values of $\omega_{\rm pe}/\omega$ and $\omega_{\rm ce}/\omega$ for extraordinary (X) wave. Here, calculation has been done for evanescent regions of $\omega_{\rm R} \gg \omega \gg \omega_{\rm UH}$ (a) and $\omega_{\rm L} \gg \omega \gg \omega_{\rm LH}$ (b).

For case of R wave propagation with 1) $\omega \gg \omega_{\rm ce}$ and $\omega \gg \omega_{\rm pe}$, 2) $\omega_{\rm ce} \gg \omega \gg \omega_{\rm ci}$, $\omega_{\rm R} \gg \omega$ and $\omega_{\rm L} \gg \omega$ (whistler wave) and 3) $\omega_{\rm L} \gg \omega$ and $\omega_{\rm ci} \gg \omega$ (Alfven wave), $L_{\rm d}$ is infinite with parallel wave number $k_{\rm r} = \omega/c$, $(\omega_{\rm pe}/c)(\omega/\omega_{\rm ce})^{0.5}$ and $(\omega_{\rm pe}/c)[\omega/(\omega_{\rm ce}\omega_{\rm ci})^{0.5}]$, respectively. Note that these waves have only parallel wave numbers, and that *e.g.*, the collisional damping length of $L_{\rm d}$ is finite for the helicon wave[14], which is categorized as a whistler wave and has a finite perpendicular wave number.

For the L wave for cases of both 1) $\omega_{\rm pe} \gg \omega_{\rm ce} \gg \omega \gg \omega_{\rm ci}$ and 2) $\omega_{\rm ce} \gg \omega_{\rm pe}$ and $\omega_L \gg \omega \gg \omega_{\rm ci}$, wave number $k_\parallel$ from eq. (5) with a collision can be approximated as follows, which shows the independence of collisionality.

$$k_\parallel^2 = -\frac{\omega\omega_{\rm pe}^2}{c^2\omega_{\rm ce}} \tag{8}$$

δ obtained from eq. (8) is different from that in eq. (3) by a large factor of $(\omega_{\rm ce}/\omega)^{0.5}$ ($\gg 1$), and $k_{\rm r}$ becomes 0. For the case of $\omega_{\rm pe} \gg \omega \gg \omega_{\rm ce}$, values of δ and $k_{\rm r}$ are the same as those in eqs. (2) and (3) because they have the same dispersion relation as given in eq. (1).

Figure 4 shows the numerical result for the dependence of enhancement factor $f_{\rm num}$ on the normalized collision frequency ν^* in the evanescent region of L waves, obtained using eq. (7). For high $\omega_{\rm pe}/\omega$ and low $\omega_{\rm ce}/\omega$ (for a given $\omega_{\rm pe}/\omega$), $f_{\rm num}$ is nearly the same as $f(\nu^*)$ in Fig. 2, as has been discussed analytically. For high $\omega_{\rm pe}/\omega$ and high $\omega_{\rm ce}/\omega$, $f_{\rm num}$ is larger than 1, which has been estimated as $(\omega_{\rm ce}/\omega)^{0.5}$ from eq. (8). Note that the case of $\omega_{\rm pe}/\omega = 100$ and $\omega_{\rm ce}/\omega = 1000$ shown in Fig. 4(c) is only applicable for the case of an ion much heavier than a proton, otherwise the approximation of $\omega \gg \omega_{\rm ci}$ in deriving eq. (7) becomes invalid. Needless to say, $f_{\rm num}$ becomes small, i.e. $\omega_{\rm ce}^{0.5}(\omega-\omega_{\rm ci})^{0.5}/\omega$ (regardless of ν^*), when ω is close to ion cyclotron resonance frequency $\omega_{\rm ci}$ ($\omega > \omega_{\rm ci}$).

In the propagating region (L wave) of $\omega_{\rm ce} \gg \omega \gg \omega_{\rm L}$ with $\omega_{\rm ce} \gg \omega_{\rm pe}$ and $\omega \gg \omega_{\rm ci}$, the damping length $L_{\rm d}$ is a large value of $(2c/\omega)(\omega_{\rm ce}/\omega_{\rm pe})^2/\nu^*$ with $k_{\rm r} = \omega/c$. This length becomes infinite (independent of ν^*) in the regions of 1) $\omega \gg \omega_{\rm ce}$ and $\omega \gg \omega_{\rm pe}$ and 2) $\omega_L \gg \omega$ and $\omega_{\rm ci} \gg \omega$ (Alfven wave) for $k_{\rm r} = \omega/c$ and $(\omega_{\rm pe}/c)[\omega/(\omega_{\rm ce}\omega_{\rm ci})^{0.5}]$, respectively.

Finally, we perform calculation for electromagnetic waves perpendicular to the magnetic field. The dispersion relation of the O wave (the perturbed electric field is parallel to the magnetic field) is the same as eq. (1) if we replace $k_\parallel$ for $k_\perp$ (perpendicular wave number). The skin depth and real wave number are again the same as those in eqs. (2) and (3) (for the case of $\omega_{\rm pe} \gg \omega$ and $\omega_{\rm pe} \gg \nu$,) (see also Figs. 1 and 2). For the case of wave propagation with $\omega \gg \omega_{\rm pe}$, real wave number (perpendicular component) $k_{\rm r}$ and damping length $L_{\rm d}$ are also the same as for the light wave ($\omega \gg \omega_{\rm pe}$) described above: $k_{\rm r} = \omega/c$ and $L_{\rm d} = (2c\omega/\omega_{\rm pe}^2)[(1+\nu^{*2})/\nu^*]$.

For the X wave (perturbed electric field is perpendicular to the magnetic field), the dispersion relation without a collision is

$$\frac{k_\perp^2 c^2}{\omega^2} = \frac{(\omega^2-\omega_{\rm R}^2)(\omega^2-\omega_{\rm L}^2)}{(\omega^2-\omega_{\rm LH}^2)(\omega^2-\omega_{\rm UH}^2)}, \tag{9}$$

where the lower and upper hybrid angular frequencies of $\omega_{\rm LH}$ and $\omega_{\rm UH}$ are defined as $1/\omega_{\rm LH}^2 = 1/(\omega_{\rm pi}^2+\omega_{\rm ci}^2) + 1/\omega_{\rm ce}\omega_{\rm ci}$ and $\omega_{\rm UH}^2 = \omega_{\rm pe}^2 + \omega_{\rm ce}^2$, respectively ($\omega_{\rm pi}$: ion plasma angular frequency). If $\omega_{\rm ce} \gg \omega \gg \omega_{\rm LH}$ with $\omega_{\rm pe} \gg \omega_{\rm ce}$, eqs. (1) (by replacing $k_\parallel$ for $k_\perp$), (2) and (3) are also satisfied if a collision is included, and another nonpropagating region of $\omega_{\rm R} > \omega > \omega_{\rm UH}$ for the case of $\omega_{\rm pe} \gg \omega_{\rm ce}$ is narrow. On the other hand, for the case of $\omega_{\rm ce} \gg \omega_{\rm pe}$, the nonpropagating region of $\omega_{\rm R} > \omega > \omega_{\rm UH}$ is narrow. In another region of $\omega_{\rm L} \gg \omega \gg \omega_{\rm LH}$, a simplified dispersion relation from eq. (9) with a collision is

$$k_\perp^2 = -\frac{\omega_{pe}^4}{c^2\omega_{ce}^2}\left(1 - i\nu^* \frac{\omega_{pe}^2}{\omega_{ce}^2}\right). \quad (10)$$

The derived skin depth is expressed as $c\omega_{ce}/\omega_{pe}^2$ (independent of ν^*), which is larger than δ_{cl} by a factor of ω_{ce}/ω_{pe} ($\gg 1$), and the real wave number k_r is a small value of $(\nu^*/2c)(\omega_{pe}^4/\omega_{ce}^3)$. Note that the above frequency condition in calculating eq. (10) is satisfied under the limited condition having suitable values of the magnetic field and plasma density with an ion much heavier than a proton.

Figure 5 shows the numerical result for the dependence of enhancement factor f_{num} ($\delta = (c/\omega_{pe})f_{num}$) on the normalized collision frequency ν^* in the evanescent region of X wave, obtained using eq. (9). Here, we assumed $\omega \gg \omega_{LH}$. Figures 5(a) and 5(b) show upper and lower frequency evanescent regions of $\omega_R > \omega > \omega_{UH}$ and $\omega_L > \omega \gg \omega_{LH}$, respectively. As has been stated, upper evanescent region is narrow, so we use the typical value of ω_{pe}/ω for each ω_{ce}/ω value for convenience as shown in Fig. 5(a). In the small ν^* region, f_{num} becomes larger with decrease in ω_{ce}/ω, which is similar to the light wave behavior for $\omega_{pe}/\omega \approx 1$ (see Fig. 2). When ω_{pe}/ω is increased (decreased) with fixed ω_{ce}/ω, which means $\omega \approx \omega_{UH}$ (ω_R), f_{num} becomes smaller (larger) in the low ν^* region. f_{num} in the high ν^* region is larger than that of light waves, and does not change with fixed ω_{ce}/ω and varied ω_{pe}/ω. On the other hand, in the lower evanescent region (Fig. 5(b)) with low ω_{ce}/ω (≤ 1), f_{num} is nearly the same as $f(\nu^*)$ in Fig. 2, for a given ω_{pe}/ω. For the case that ω approaches a lower hybrid resonance frequency of ω_{LH} ($\omega > \omega_{LH}$) with $\nu^* \approx 0$, f_{num} becomes a small value of $(\omega^2 - \omega_{LH}^2)^{0.5}\omega_{UH}/\omega\omega_{pe}$.

In the X wave propagation region of $\omega \gg \omega_{pe}$ and $\omega \gg \omega_{ce}$, infinite damping length L_d and $k_r = \omega/c$ are obtained. When $\omega_{ce} \gg \omega_{pe}$ is satisfied, $k_r = \omega/c$ with $L_d = (2c/\omega)(\omega_{ce}/\omega_{pe})^2/\nu^*$ for the case of $\omega \gg \omega_{LH}$ and $\omega_{UH} \gg \omega \gg \omega_L$ (propagation region), and this propagating region becomes small when $\omega_{pe} \gg \omega_{ce}$ is satisfied. When we consider a low-frequency mode of $\omega_{LH} \gg \omega$ (propagating region) under a condition of $\omega_{pe} \gg \omega_{ce}$, we have $k_r = (\omega_{pe}/c)[\omega/(\omega_{ce}\omega_{ci})^{0.5}]$ (Alfven wave) with $L_d = 2c\omega_{pe}\omega_{ci}^{0.5}/\omega\omega_{ce}^{1.5}\nu^*$, and $k_r = \nu^*\omega\omega_{pe}^2/c\{2\omega_{ce}^3\omega_{ci}[-1 + (1+\nu^{*2})^{0.5}]\}^{0.5}$ and $L_d = (c/\omega\omega_{pe}^2)\{2\omega_{ce}^3\omega_{ci}/[-1 + (1+\nu^{*2})^{0.5}]\}^{0.5}$ under a condition of $\omega_{ce} \gg \omega_{pe}$ and $\omega_L \gg \omega$. Here, we assumed $\omega_{LH}^2 \approx \omega_{ce}\omega_{ci}$ for the low-frequency mode ($\omega_{LH} \gg \omega$).

In conclusion, analytic formulae of the skin depth δ and real wave number k_r of electromagnetic evanescent waves, with and without magnetic field in plasma in which collisions occur, have been derived from numerical results. A damping length L_d and real wave number k_r have also been evaluated under propagating conditions. For evanescent waves with $\omega_{pe} \gg \omega$ and $\omega_{pe} \gg \nu$ in most cases, δ is expressed as $\delta_{cl} = c/\omega_{pe}$ multiplied by the collision term $f(\nu^*)$, and k_r as $g(\nu^*)\omega_{pe}/c$, as described in eq. (3). For the L (conditions of $\omega_{pe} \gg \omega_{ce} \gg \omega \gg \omega_{ci}$, or $\omega_{ce} \gg \omega_{pe}$ and $\omega_L \gg \omega \gg \omega_{ci}$) and X (limited condition of $\omega_{ce} \gg \omega_{pe}$ and $\omega_L \gg \omega \gg \omega_{LH}$) waves, $\delta (= (c/\omega_{pe})f_{num})$ is independent of ν^* and f_{num} is expressed as $(\omega_{ce}/\omega)^{0.5}$ ($\gg 1$) and ω_{ce}/ω_{pe} ($\gg 1$), respectively, while real wave number k_r is zero (L wave) and $(\nu^*/2c)(\omega_{pe}^4/\omega_{ce}^3)$, respectively. When ω approaches ω_{pe} in the low ν^* region, f_{num} becomes large for light, O, R and X waves ($\omega \gg \omega_{ce}$). On the contrary, if ω is near resonance frequencies of ω_{ce} (R wave), ω_{ci} (L wave), ω_{UH} (X wave) and ω_{LH} (X wave) in the evanescent region, f_{num} becomes smaller than unity.

1) M. J. Kofoid and J. M. Dawson: Phys. Rev. Lett. **17** (1966) 1086.
2) T. R. Soldatenkov: Nucl. Fusion **10** (1973) 69.
3) K. Minami, K. P. Singh, M. Masuda and K. Ishii: Phys. Rev. Lett. **33** (1974) 740.
4) V. A. Godyak: *Soviet Radio Frequency Discharge Research* (Delphic Associates, Inc., Falls Church, Virginia, 1986).
5) J. Hopwood: Plasma Sources Sci. & Technol. **1** (1992) 109.
6) M. M. Turner: Phys. Rev. Lett. **71** (1993) 1844.
7) S. Shinohara, S. Takechi and Y. Kawai: submitted to Jpn. J. Appl. Phys.
8) N. Hiraki, K. Nakamura, Y. Nakamura and S. Itoh: Jpn. J. Appl. Phys. **20** (1981) 769.
9) R. D. Jones: Phys. Rev. Lett. **51** (1983) 1269.
10) G. Bekefi: *Radiation Processes in Plasmas* (John Wiley & Sons, Inc., New York, 1966).
11) T. H. Stix: *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).
12) K. Miyamoto: *Plasma Physics for Nuclear Fusion* (The MIT Press, Cambridge, 1988).
13) A. Anders: *A Formulary for Plasma Physics* (Academic-Verlag, Berlin, 1990).
14) F. F. Chen: Plasma Phys. Control. Fusion **33** (1991) 339.