the onset criterion used. Therefore, the influence of the horizontal elongation on the onset of tearing modes can be described by the safety factor alone.

In conclusion, passive feedback control of axisymmetric modes in approximately elliptic-cross-section plasmas allowed $b/a \gtrsim 0.75$. For stability without control, $b/a \gtrsim 0.85$. With a restricted aperture, $b/a \gtrsim 0.95$. These limits are predicted by rigid-shift calculations in which flux conservation and experimental circuit parameters are accounted for.

Confirmation of the shape predicted by equilibrium calculations is obtained from field perturbations produced by tearing modes. This implies that only the safety factor determines the instability onset.

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REFERENCES

[1] KADOMTSEV, B.B., Comments on Plasma Physics and Controlled Fusion 2 (1976) 75.

[2] SOLEVEV, L.S., SHAFRANOV, V.D., YURCHENKO, E.I., Nucl. Fusion Supplement (1969) 25.

[3] KING, R.E., WOOTTON, A.J., ROBINSON, D.C., in Fusion Technology (Proc. 8th Symp. Commission of the European Communities) (1974) 59.

[4] MUKHOVATOV, V.S., SHAFRANOV, V.D., Nucl. Fusion 14 (1974) 263.

[5] THOMAS, C.L., Computational Methods in Classical and Quantum Physics, Advanced Publications Ltd., London (1976).

[6] ZAKHAROV, L.E., SHAFRANOV, V.D., Sov. Phys.-Tech. Phys. 18 (1973) 151.

[7] REBHAN, E., SALAT, A., Nucl. Fusion 17 (1977) 251.

[8] FUKUYAMA, A., SEKI, S., MOMOTA, H., ITATANI, R., Jap. J. Appl. Phys. 14 (1975) 871.

[9] GREENE, J.M., JOHNSON, J.L., WEIMER, K.E., Phys. Fluids 14 (1971) 671.

[10] CROW, J.E., KILLEEN, J., ROBINSON, D.C., in Controlled Fusion and Plasma Physics (Proc. 6th Europ. Conf.) 1, USSR Academy of Sciences, Moscow (1973) 269.

[11] TOYAMA, H., MAKISHIMA, K., KANEKO, H., NOGUCHI, M., SOSHIKAWA, S., Phys. Rev. Lett. 37 (1976) 18.

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A PROPOSED METHOD OF DETERMINING THE CURRENT DENSITY PROFILE BY UTILIZING ORDINARY AND EXTRAORDINARY WAVES

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ABSTRACT. A proposal for determining the current density profile of tokamaks by the use of ordinary and extraordinary waves is presented. Transmission paths of both waves are taken along a constant major radius. Using microwave interferometry, the phase shifts, which depend on the magnetic field, give the current density profile. An estimate of the plasma density distribution is also discussed.

The poloidal field, which is related to plasma confinement, is very important in nuclear-fusion research, but the current profile has not been measured directly. In a large-scale tokamak, information on the current profile can only be obtained indirectly, e.g. from a measurement of the electron temperature and the value of Z_{eff}. Recently, a measurement of the direction of the magnetic field by laser light scattering has been proposed and now carried out in the DITE tokamak [1]. This method uses the cyclotron modulation of the scattered light spectrum.

In this paper, we propose a method of estimating the current profile by measuring the phase shift of the ordinary (O) and the extraordinary (X) microwaves, when the density distribution function is known.

When the waves propagate through the plasma in a tokamak, the angle (θ) between the direction of the magnetic field line and that of wave propagation is changed owing to the poloidal field.

If the collision and the thermal terms are neglected, the refractive indices of the O- and X-waves are expressed in the following form [2]:

$$\mu_O = \left[1 - \frac{\omega_p^2}{\omega^2} \middle/ \left\{ 1 - \frac{\dfrac{\omega_{ce}^2}{\omega^2}\sin^2\theta}{2\left(1 - \dfrac{\omega_p^2}{\omega^2}\right)} \right. \right.$$

$$+\sqrt{\frac{\frac{\omega_{ce}^4}{\omega^4}\sin^4\theta}{4\left(1-\frac{\omega_p^2}{\omega^2}\right)^2}+\frac{\omega_{ce}^2}{\omega^2}\cos^2\theta}\Bigg\}\Bigg]\Bigg]^{\frac{1}{2}} \qquad (1)$$

$$\mu_X = \left[1-\frac{\omega_p^2}{\omega^2}\Bigg/\left\{1-\frac{\frac{\omega_{ce}^2}{\omega^2}\sin^2\theta}{2\left(1-\frac{\omega_p^2}{\omega^2}\right)}\right.\right.$$

$$-\sqrt{\frac{\frac{\omega_{ce}^4}{\omega^4}\sin^4\theta}{4\left(1-\frac{\omega_p^2}{\omega^2}\right)^2}+\frac{\omega_{ce}^2}{\omega^2}\cos^2\theta}\Bigg\}\Bigg]\Bigg]^{\frac{1}{2}} \qquad (2)$$

respectively, where ω is the applied frequency, ω_p the electron plasma frequency, and ω_{ce} the electron cyclotron frequency. Using these refractive indices, we can write the phase shifts of the O- and X-waves in a plasma of dimension 2L as follows:

$$\Delta\Phi_O = \frac{2\pi}{\lambda}\int_{-L}^{L}(1-\mu_O(x))\,dx \qquad (3)$$

$$\Delta\Phi_X = \frac{2\pi}{\lambda}\int_{-L}^{L}(1-\mu_X(x))\,dx \qquad (4)$$

where λ is the wavelength of the applied frequency.

We assume that

(i) the conditions $\alpha \ll 1$, $\alpha/(1-\beta^2) \ll 1$ are satisfied, where $\alpha = \omega_p^2/\omega^2$ ($= n_e e^2/m_e\epsilon_0\omega^2$, n_e: electron density, m_e: electron mass, e: electron charge, ϵ_0: dielectric constant in the vacuum), $\beta = \omega_{ce}/\omega$,

(ii) both O- and X-waves take the same path.

(iii) the density distribution along the transmission path is expressed as

$$n_e(x) = n_c\{1-(x/L)^2\}^k \qquad (5)$$

(n_c is the value of the peaked density.)

When θ is nearly $\pi/2$ radians, the refractive indices of the O- and the X-waves, to first order in the ϕ^2 term and second order in α, are

$$\mu_O = 1-\frac{\alpha}{2}-\frac{\alpha^2}{8}+\frac{\alpha\phi^2}{8} \qquad (6)$$

$$\mu_X = 1-\frac{\alpha}{2(1-\beta^2)}-\frac{1+4\beta^2}{8(1-\beta^2)^2}\alpha^2-\frac{\alpha\phi^2}{8(1-\beta^2)} \qquad (7)$$

where $\phi = \pi - 2\theta$ is satisfied.

Using Eqs (3), (5), and (6), we have

$$\Delta\Phi_O = \frac{2\pi}{\lambda}\int_{-L}^{L}\left[\frac{\alpha_c}{2}\left\{1-\left(\frac{x}{L}\right)^2\right\}^k+\frac{\alpha_c^2}{8}\left\{1-\left(\frac{x}{L}\right)^2\right\}^{2k}\right.$$

$$\left.+\frac{\alpha_c}{8}\left\{1-\left(\frac{x}{L}\right)^2\right\}^k\phi^2(x)\right]dx$$

$$= \frac{\pi\alpha_c L}{\lambda}\left\{B\left(\frac{1}{2},k+1\right)+\frac{\alpha_c}{4}B\left(\frac{1}{2},2k+1\right)\right\}-\frac{\pi L}{2\lambda}\langle\alpha\phi^2\rangle \qquad (8)$$

where

$$\langle A\rangle = \frac{1}{2L}\int_{-L}^{L}A(x)\,dx$$

$\alpha_c = n_c e^2/m_e\epsilon_0\omega^2$ and

$$B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}dt \quad \text{(beta function)}$$

Similarly, $\Delta\Phi_X$ is obtained from Eqs (4), (5), and (7), when β is nearly independent of the position:

$$\Delta\Phi_X = \frac{\pi\alpha_c L}{\lambda(1-\beta^2)}\left\{B\left(\frac{1}{2},k+1\right)+\frac{\alpha_c(1+4\beta^2)}{4(1-\beta^2)}\right.$$

$$\left.\times B\left(\frac{1}{2},2k+1\right)\right\}+\frac{\pi L}{2\lambda(1-\beta^2)}\langle\alpha\phi^2\rangle \qquad (9)$$

Let's calculate the $\langle\alpha\phi^2\rangle$ term in the case where the current density is expressed as $j(r) = j_0\{1-(r/a)^2\}^m$ (a is the plasma current radius, m is an integer). Figure 1 shows the cross-section of the cylindrical plasma. The x-axis runs parallel to the propagation of the waves ($\vec{K}$), and the z-axis is along the toroidal field ($\vec{B}_t$). The poloidal field ($\vec{B}_p$) has the x-component

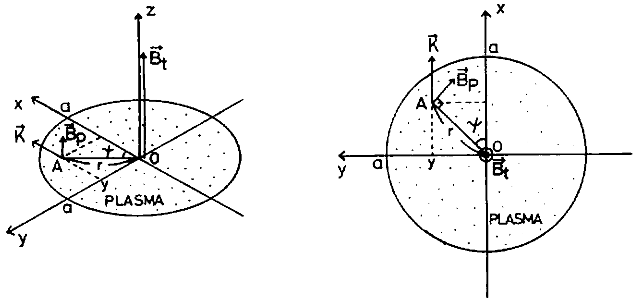

FIG.1. Cross-section of cylindrical plasma and variables used to calculate the $\langle\alpha\phi^2\rangle$ term in the circular tokamak. $\vec{B}_t$: toroidal field, $\vec{B}_p$: poloidal field, $\vec{K}$: propagation of waves, a: plasma radius.

$B_p \sin\psi$ and the y-component $-B_p \cos\psi$ at the point A $(r\cos\psi, r\sin\psi, 0)$ in the (x, y, z) co-ordinate system.

When $B_t \gg B_p$ is satisfied,

$$\cos\theta = \frac{\vec{B}\cdot\vec{K}}{|\vec{B}|\cdot|\vec{K}|} \cong \frac{B_p}{B_t}\sin\psi \qquad (10)$$

at the point A in Fig.1. Using

$$\phi^2 \cong 4\cos^2\theta \cong 4\left(\frac{B_p}{B_t}\sin\psi\right)^2 \qquad (11)$$

we have

$$\langle\alpha\phi^2\rangle = \frac{2}{L}\int_{-L}^{L}\alpha(x)\left(\frac{B_p}{B_t}\right)^2\frac{y^2}{x^2+y^2}\,dx \qquad (12)$$

Making use of $L = \sqrt{a^2 - y^2}$, we finally obtain:

$$\langle\alpha\phi^2\rangle = \frac{\alpha_c}{2}\left|\frac{\mu_0 j_0 a^2 y}{B_t(m+1)}\right|^2 \sum_{\substack{j=i+i' \\ 1\le i\le m+1 \\ 1\le i'\le m+1}} \frac{{}_{m+1}C_i\,{}_{m+1}C_{i'}}{(-a^2)^j}$$

$$\times\sum_{n=0}^{j-2}{}_{j-2}C_n\,y^{2n}(a^2-y^2)^{j-n-2}B\left(j-n-\frac{3}{2},k+1\right)$$

$$= 2\alpha_c\left(\frac{y}{q_aR}\right)^2\sum_{\substack{i,i' \\ j=i+i'}}(-1)^j\,{}_{m+1}C_i\cdot{}_{m+1}C_{i'}$$

$$\times\sum_{n}{}_{j-2}C_n\left(\frac{y}{a}\right)^{2n}\left(1-\frac{y^2}{a^2}\right)^{j-n-2}B\left(j-n-\frac{3}{2},k+1\right) \qquad (13)$$

where q_a is the safety factor at the boundary r = a and R is the major radius.

For example,

$$\langle\alpha\phi^2\rangle = 2\alpha_c\left(\frac{y}{q_aR}\right)^2 B\left(\frac{1}{2},k+1\right) \qquad (m=0) \qquad (14)$$

$$\langle\alpha\phi^2\rangle = 2\alpha_c\left(\frac{y}{q_aR}\right)^2\left|\left(2-\frac{y^2}{a^2}\right)^2 B\left(\frac{1}{2},k+1\right) - 2\left(1-\frac{y^2}{a^2}\right)\left(2-\frac{y^2}{a^2}\right)B\left(\frac{3}{2},k+1\right) + \left(1-\frac{y^2}{a^2}\right)^2 B\left(\frac{5}{2},k+1\right)\right| \qquad (m=1) \qquad (15)$$

Figure 2 shows the dependence of $L\langle\alpha\phi^2\rangle$ on y/a with k = 0, 1, 2, and m = 0, 1, 2.

When we express $\langle\alpha\phi^2\rangle$ as $\alpha_c\phi_\alpha^2$, α_c, 2L, and n_c are derived from Eqs (8) and (9):

$$\alpha_c = \frac{4B\left(\frac{1}{2},k+1\right)\{(1-\beta^2)\Delta\Phi_X-\Delta\Phi_O\}-2\phi_\alpha^2\{\Delta\Phi_O+(1-\beta^2)\Delta\Phi_X\}}{B\left(\frac{1}{2},2k+1\right)\left|\frac{1+4\beta^2}{1-\beta^2}\Delta\Phi_O-(1-\beta^2)\Delta\Phi_X\right|} \qquad (16)$$

$$2L = \frac{2\lambda\Delta\Phi_O}{\pi\alpha_c}\,\frac{1}{B\left(\frac{1}{2},k+1\right)+\frac{\alpha_c}{4}B\left(\frac{1}{2},2k+1\right)-\frac{\phi_\alpha^2}{2}} \qquad (17)$$

$$n_c = \frac{m_e\epsilon_0}{e^2}\,\omega^2\alpha_c \qquad (18)$$

When the density distribution function is known (the values of n_c, k, L are known), the values of $\langle\alpha\phi^2\rangle$ are obtained by Eqs (8) and (9) in one path of the O- and the X-waves. We can obtain the values of m from those of $\langle\alpha\phi^2\rangle$, using Eq.(13). With many channels the current profile is obtained more accurately.

The accuracy of this method is limited by the higher terms of α and ϕ^2, and the horn aperture.

In the case of a non-circular tokamak, Eqs (8) and (9) are used, but the $\langle\alpha\phi^2\rangle$ term must be calculated according to the current profile. Nevertheless, with many channels of the waves, the current profile can be estimated from the values of $\langle\alpha\phi^2\rangle$.

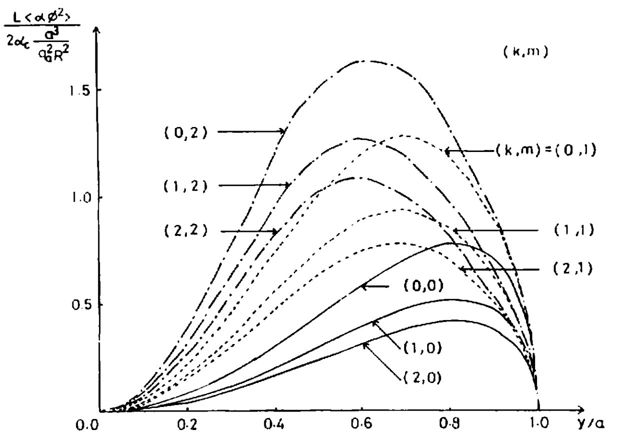

FIG.2. $L\langle\alpha\phi^2\rangle$ as a function of the position y/a with k = 0, 1, 2 and m = 0, 1, 2.

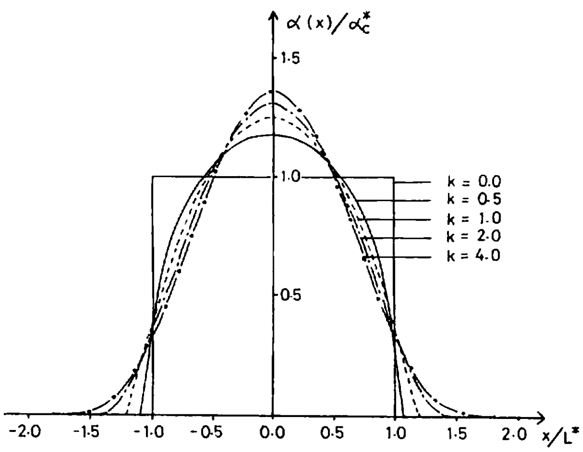

FIG.3. Plot of $\alpha(x)/\alpha_c^*$ versus x/L^* for various values of k. The value of k is changed from 0.0 to 4.0 in this figure, as the condition $0.5 \leqslant k \leqslant 2.0$ is satisfied in most of the present tokamaks.

Although the current profile is derived from the density distribution, the latter can be derived if the current profile is known, by using Eqs (13), (16), (17) and (18).

If the aspect ratio and the safety factor are large, the value of $\langle\alpha\phi^2\rangle/\langle\alpha^2\rangle$ becomes small and ϕ_α^2 may be regarded as vanishing. For example, if the aspect ratio is 4, k = 1, m = 1, $q_a = 8$, $\alpha_c = 0.1$, and y = a/2, $\langle\alpha\phi^2\rangle$ and $\langle\alpha^2\rangle$ become 1.7×10^{-4} and 5.3×10^{-3}, respectively. In this case, 2L and n_c are calculated, only assuming the value of k along the one path even when the cross-section is non-circular (this yields information on the elongation). Figure 3 shows $\alpha(x)/\alpha_c^*$ versus x/L^* for various values of k, when the values of $\Delta\Phi_O$ and $\Delta\Phi_X$ are fixed. In the case of a flat distribution, α_c is expressed as α_c^* and L as L^*. In most of the present tokamaks, the value of k lies between 0.5 and 2.0 (e.g. k = 1.5 in the stationary stage for T-10 [3], k = 0.6 – 1.0 for TFR [4], k = 1.0 for Doublet IIA [5], and k = 1.0 – 2.0 for JFT-2 [6]). This figure shows that the value of L changes about ± 10% and α_c ± 5% when the condition $0.5 \leqslant k \leqslant 2.0$ is satisfied.

We can also obtain more information on the current profile with an arbitrary cross-section even when the conditions (i), (ii), (iii) are not satisfied. The current density profile is determined from Eqs (1) to (4) with more channels of the microwave interferometry than the number of the free parameters of the assumed current density profile. For example, if we assume a current density profile of the form

$$j(x, y) = j_0 \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^m$$

(the cross-section is in the x, y-plane), more than three channels are necessary to determine the three parameters a, b, m.

In this letter, it is shown that the relation between current and density profiles can be calculated by measuring the phase shifts of O- and X-waves. This method is applied to a determination of the current profile.

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REFERENCES

[1] SHEFFIELD, J., Plasma Phys. 14 (1972) 385; CAROLAN, P.G., Plasma Phys. 19 (1977) 757; FORREST, M.J., CAROLAN, P.G., PEACOCK, N.J., Nature 271 (1978) 718.

[2] HEALD, M.A., WHARTON, C.B., Plasma Diagnostics with Microwaves, John Wiley (1965); HUDDLESTONE, R.H., LEONARD, S.L., Plasma Diagnostic Techniques, Academic Press (1965).

[3] BERLIZOV, A.B., BOBROVSKIJ, G.A., BAGDASAROV, A.A., VASIN, N.L., VERTIPOROKH, A.N., et al., in Plasma Physics and Controlled Nuclear Fusion Research 1976 (Proc. 6th Int. Conf. Berchtesgaden, 1976) 1, IAEA, Vienna (1977) 3.

[4] EQUIPE TFR, ibid., 1, IAEA, Vienna (1977) 69.

[5] FREEMAN, R.L., ADCOCK, S.J., BROOKS, N.H., DEBOO, J.C., FISHER, R.K., et al., A Comparison of Circular and Elliptic Plasmas in Doublet IIA, General Atomic Report GA-A13781 (1976).

[6] FUJISAWA, N., FUNAHASHI, A., KUNIEDA, S., MAENO, M., SUZUKI, N., et al., in Plasma Physics and Controlled Nuclear Fusion Research 1974 (Proc. 5th Int. Conf. Tokyo, 1974) 1, IAEA, Vienna (1975) 3.

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ON THE RAY-HORA THEORY OF STOPPING POWER OF PLASMA ELECTRONS

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ABSTRACT. It is shown that the Ray-Hora theory describes the behaviour of plasma stopping power qualitatively well only in those cases where the test ion speed is much higher than the thermal speed of the plasma electrons. Consequently, the conclusions of this theory concerning significant reheating effects in laser-driven fusion plasmas are questionable.

Let us consider the energy transfer from energetic test ions of charge Ze to plasma electrons in thermal equilibrium at a temperature T (in energy units). Basically, one has to solve Boltzmann- or Fokker-Planck-type equations to find the relaxation of the energy distribution function of the test ions starting from a sharp initial distribution. However, as long as the ion is sufficiently energetic so that $E \gg T$, the energy dispersion is negligible compared to E itself. It is then meaningful to discuss the problem in terms of the average energy loss by the ion per unit distance traversed, $-dE/dz$, i.e. the stopping power of the plasma electrons.

Ray and Hora [1, 2] have recently presented a theory which leads to a stopping power given by

$$-dE/dz = (Ze\omega_p/V)^2 \ln [V^2 (mT)^{\frac{1}{2}}/Ze\omega_p] \qquad (1)$$

where V is the speed of the ion, m the electron mass, and $\omega_p = (4\pi ne^2/m)^{1/2}$ is the plasma frequency, with n the electron number density. The stopping power (1) increases with T for a given E or V. It also keeps increasing as E approaches the thermal energy from above. These features are very hard to understand physically. Thus, formula (1) leads to a thermalization range narrower than more conventional values by quite a few orders of magnitude at sufficiently high T. This, in turn, results in a very strong re-heating effect by fusion-produced nuclei which significantly increases reaction gains in an inertially confined hot plasma [3].

In view of such an important consequence, we have critically examined the validity of formula (1). Our conclusion is that the use of formula (1) cannot be justified for the discussion of the re-heating effect in a hot plasma. Consequently, Ray and Hora's conclusions on the reaction gains [3] are open to question. It is the purpose of the present letter to report on this point.

The derivation of formula (1) by Ray and Hora [1] is identical with the derivation of the stopping power of ordinary matter often found in text books of nuclear physics, e.g. Segré [4]. It is then immediately clear that the validity of formula (1), if existing at all, is subject to the condition that the ion speed be much higher than the average electron thermal speed, i.e. $x \gg 1$ where

$$x = V(m/2T)^{\frac{1}{2}} \qquad (2)$$

Ray and Hora's choice of the minimum impact parameter,

$$b_{min} = Ze^2/mV^2 \qquad (3)$$

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